ON THE STATISTICAL DISTRIBUTION OF THE LARGEST LENGTHS OF FISH

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I. INTRODUCTION

A confusing situation that most often confronts a biologist is the observation that a deterministic growth equation like the von Bertalanffy growth equation is generally obeyed when a relatively large number of fish are studied and yet the same equation fails miserably when applied to an individual fish growing over time.

To understand this confusion, it must be remembered that the von Bertalanffy equation

$$L_t = L_\infty (1 - e^{-k(t - t_0)})$$

(where L_t is the length at age t; L_∞ is the mean length the fish would have reached if they were to grow to a very old age; k is a growth coefficient; and t_o is the "age" of the fish at length zero) is a deterministic equation; and as such, indicates that if the parameters L_∞, k and t_o are fixed, or are functionally determined, then for a particular t value, there is only one value for L_t. Thus we can imagine that if we have two identical fish (i.e., of the same species and age and are initially of the same length) and we let these two fish grow in exactly the same environment, then after a certain period of time, the two fish would have attained exactly the same length. Yet, observations of growth of individual fish growing under controlled conditions do not bear this out. Rather, the most likely event to happen is to have the fish grow to different lengths for the same length of time.

Clearly, fish growth is not deterministic in nature. Rather, fish growth is better represented as a process whose development over time is governed by probabilistic laws; that is, as a stochastic process.

A stochastic model for growth may be taken as

$$Y(t) = L(t) + \xi(t)$$

where Y(t) is the actual length of the fish at time t,

L(t) is the length that the fish would have attained if it grew according to the deterministic equation like the von Bertalanffy equation, and

$\xi(t)$ is a random variable, sometimes referred to as a random noise or a perturbation, which represents all other influences affecting growth that cannot be exactly accounted for nor determined as in L(t).

The usual assumptions imposed on the random variable $\xi(t)$ are that

$$E[\xi(t)] = 0$$

$$Var[\xi(t)] = \sigma^2 \text{ (a constant).}$$

Thus, under this stochastic model for growth, the actual length of the fish, Y(t), is a random variable whose distribution is determined by the distribution of $\xi(t)$ and whose values for fixed t values will vary according to this probability distribution.

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An important consideration is the fact that

$$E[Y(t)] = L(t)$$

that is, the average in the long run of the length values of the fish of the same species and age is provided by the length value obtained from the deterministic length equation; say, the von Bertalanffy growth equation for length. Thus, if a large number of fish of the same species and age are studied, it is the average characteristics of these large group of fish that is predicted by the von Bertalanffy equation.

One of the parameters affecting the value of $L(t)$, the length predicted by the von Bertalanffy equation, is L_∞, the mean length the fish would have attained if the fish were to grow to a very old age. Various techniques of estimating L_∞ are found in the literature, the majority of which are based on a linear transformation of the von Bertalanffy equation. Attempts to estimate L_∞ independently of the von Bertalanffy equation were provided by such rules of thumb as: take L_∞ to be 2/3 the biggest length measurement ever recorded for the given species; or take L_∞ to be the average length of a number of very old fish.

This paper is a further attempt at finding an estimator for L_∞ that is independent of the assumed underlying deterministic equation governing the growth of fish. The technique is based on the observation that various surveys have reported different values for the length of the biggest fish of a given species, indicating that the longest length of fish of a given species is not a fixed quantity but a random variable which takes on different values according to some probabilistic law.

Thus, inorder to identify a reliable estimator for L_∞ for fish of a given species, it is important that the statistical distribution of the longest lengths of that particular fish be first established.

The distribution of the longest lengths of Rastrelliger brachysoma (local name: hasa-hasa) caught from the Visayan Sea from January to December 1984 is studied via the theory of extreme values developed by Gumbel (1954) which aims to explain observed extremes arising in samples of given sizes, or valid for a given period, or length, area, or volume, and to forecast extremes that may be expected to occur within a certain sample; time, area, etc.

The application of the theory of extreme values starts with the fulfillment of the following conditions:

1) The variables (length, in our case) are continuous.
2) All the samples from where the extreme lengths are drawn have a constant distribution with fixed parameters.
3) The extreme lengths are taken from independent samples.

Note that length measures are continuous measures, the distribution of fish lengths may be safely assumed normal with fixed parameters for a particular species in a particular area, and the various surveys from which the extreme lengths were obtained may be treated as independent samples. Thus, the study of the distribution of the extreme length values is a natural application of the theory of extreme values.

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II. THEORETICAL CONSIDERATIONS

1. Exact Distribution of Extreme Values

Consider the original set of length measurements. $F(x)$ is the probability that any observed length is less than a specified value, x.

Consider also the set of maximum lengths drawn from the original observations. Let $\Phi_n(x)$ be the probability that the largest value is less than a given length x. Therefore

$$\Phi_n(x_n) = F^n(x_n)$$

whose derivative

$$\theta_n(x_n) = nF^{n-1}(x_n)f(x_n)$$

is the distribution of the largest value among n independent observations.

Similarly, the distribution of the smallest values among n independent measurements is

$$\Phi_{1n}(x_1) = 1 - [1 - F(x_1)]^n$$

and

$$\theta_{1n}(x_1) = n[1 - F(x_1)]^{n-1} f(x_1).$$

Here x_1 is the smallest value and x_n is the largest value.

2. Asymptotic Distribution of Extreme Values

Even if the initial distribution of the sample is unknown, the knowledge of the type of distribution is enough to determine the distribution of the extreme values by deriving its asymptotic distribution. An asymptotic distribution of a random variable (maximum length, in our case) is any distribution that is approximately equal to the actual distribution of the extreme lengths for a large sample size.

If the variable is infinite to the right, then its cumulative distribution function F(x) approaches 1 as quickly as the exponential function. Variables with this characteristic have asymptotic distributions which belong to the exponential type. Variables which are initially distributed as exponential, normal, chi square, logistic and the log-transformed normal belong to the exponential group. Under this type of asymptotic distribution, all moments exist but not all distributions with existing moments belong to this class. The distribution of extreme values belonging to this type is

$$\Phi(y) = e^{-e^{-y}}, \quad \theta(y) = e^{-y - e^{-y}}, \quad -\infty < y < \infty \qquad \textbf{(Equation 1)}$$

with the reduced variate $y = \alpha(x - u)$ **(Equation 2)**

and where x is the variable belonging to the exponential type (and is continuous to the right).

$1/\alpha$ is a measure of dispersion which gives the scale of measure applicable to the observed value of x to that of the reduced variate y.

u is an average (specifically, the mode for the exponential type) of the extreme value distribution.

3. The Extremal Probability Paper

A simple tool for the study of extreme values is the probability paper which gives a simple graphical method of testing the fit between theory and observations.

Let x be a continuous variate, unlimited in both directions, for which a linear reduction (equation 2):

$$x = u + y/\alpha \quad \Longrightarrow \quad y = \alpha(x - u)$$

exists, where u is a certain average and $1/\alpha$ a certain measure of dispersion.

A probability paper is a rectangular grid where the observed variate x is plotted on one axis, and the reduced variate y is plotted on the other axis.

Further, note that if F(x) is the probability distribution of the variate x and $\Phi(y)$ is the probability distribution of the reduced variate y, then

$$\Phi(y) = F(x).$$

Thus, the probability paper also includes the probability $\Phi(y) = F(x)$ plotted on a scale parallel to the scale of y.

If the theory holds (i.e., that the observations x are distributed according to F(x)), then the observations plotted on the probability paper should fit the straight line given by

$$x = u + y/\alpha \quad .$$

An extremal probability paper may be constructed using an ordinary graphing paper by using the fact that its horizontal and vertical axes have linear scales. The length units are arranged along the vertical axis; while the reduced variate y and the probabilities are plotted independently along the horizontal axis.

Since the scale of the probabilities is nonlinear, this axis is constructed based on the linear scale of y. Values for $\Phi(y)$ are purposely selected for quick interpretation and are computed and positioned using the formula for $\Phi(y)$ in equation 1. (Here recall that $\Phi(y) = F(x)$). y values in the graph range from -2 to 7 since $\Phi(y)$ of points outside this interval converge toward 0 and 1, respectively.

If the normal probability paper is used instead, the most obvious difference is the <u>curve</u> of expected extremes. An extremal probability paper clearly shows a straight line of such values. However, the scatter of the observations around the theoretical curve/line in both cases seems to be the same.

The line of expected extremes in the extremal probability paper is a straight line because of the linear scale of the reduced variate y (along the horizontal axis) from where the location of the probability values are based and the assumption of the existence of

$$x = u + y/\alpha \qquad \text{(Equation 2A)}$$

which is another way of writing equation 2.

III. <u>METHODOLOGY</u>

1. <u>Plotting the Observations</u>

After the maximum lengths are removed from their respective sets of observations, these values are arranged according to size. If one is studying extreme largest values, the extremes are arranged in increasing order. Otherwise, arrange the values from highest to lowest.

The observed lengths are then plotted on an extremal probability paper using plotting positions which are values computed from the order of the observations. A plotting position may be interpreted as the cumulative probability assigned to the mth observation.

Gumbel prefers to use the plotting position, m/(n+1), for the mth observation and n is the total number of extreme values. This choice of plotting position enables the plotting of both the smallest and the largest of extreme values in the probability paper. Alternative positions lose either of the two mentioned extremes.

An observation is plotted using its length as the ordinate and its plotting position as the abscissa.

2. Estimation of Parameters

After arranging the lengths from smallest to highest, the sample mean $\bar{x}$, and the sample standard deviation S_x, of the measurements are computed using these formulas:

$$\bar{x} = \frac{\sum_{m=1}^{n} x_m}{n} \quad \text{and} \quad S_x = \sqrt{\frac{\sum_{m=1}^{n} x_m^2 - \frac{\left(\sum_{m=1}^{n} x_m\right)^2}{n}}{n}} .$$

These values are used to get the parameters u and $1/\alpha$ needed for the fitting of the theoretical line (equation 2).

Two sets of estimates of u and $1/\alpha$ are obtained with the application of the classical least squares method to the vertical (using length values) and horizontal (using values of the reduced variate) differences between the points along the fitted line and the observed measurements. These estimates involve rather troublesome computations which may be reduced by getting the geometric means of the parameters obtained from the vertical and horizontal differences stated above. Anyway, separate computations of all the estimates show that the values do not largely differ from each other.

The parameters $1/\alpha$ and u are computed as follows:

$$1/\alpha = \frac{S_x}{\sigma_n} \qquad \text{and} \qquad u = \bar{x} - \bar{y}_n/\alpha .$$

where $\bar{x}$ and S_x are the same as that explained at the beginning of this section.

$\bar{y}_n$ and σ_n are the mean and standard deviation, respectively, of the plotting positions, m/(n+1). These two quantities are used to solve the parameters u and $1/\alpha$ as can be seen above. $\bar{y}_n$ and σ_n are fixed for a specific n and a table for such values is given in Gumbel's monograph. Likewise, a computer program which includes the computation of $\bar{y}_n$ and σ_n is provided at the end of this article. These values are not statistics (since they don't depend on the observations but on plotting positions) nor are they purely population values (since they are influenced by the sample size n).

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3. Fitting the Straight Line

Having computed the parameters u and $1/\alpha$, the theoretical straight line $x = u + y/\alpha$ (equation 2), can now be fitted to the observations. By selecting a few length values, their corresponding y value is computed using equation 2. Again, with length as the ordinate and the reduced variate as the abscissa, plot these points over the observations in the extremal probability paper then connect all theoretical points to make the line of expected extremes.

A good fit between the observations and the theoretical line implies that the statistical theory of extreme values holds true. In this way, the line of expected extremes enables one to predict the occurrence of a maximum length and its probability under constant environmental conditions.

4. Control Curves

The control curves provide a graphical way of testing the goodness of fit of the theoretical straight line to the actual observations. To construct control curves, first compute the standard error of the mth reduced variate using this equation:

$$\sqrt{n}\ \sigma(y_m) = \sqrt{\Phi(y_m)[1 - \Phi(y_m)]/\ \theta(y_m)} \qquad \text{(Equation 3)}$$

where $\Phi(y_m)$ is the frequency of the mth extreme length computed from equation 1, and

$\theta(y_m)$ is the first derivative of $\Phi(y_m)$; $\theta(y_m)$ is computed from equation 1.

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With the value obtained in equation 3 as the numerator, solve for the standard error of the mth observation, x_m, with the equation below:

$$\sigma(x_m) = \sqrt{n}\ \sigma(y_m) / (\sqrt{n}\ \alpha) \qquad \text{(Equation 4)}$$

where $1/\alpha$ is the same parameter previously defined, and

n is the number of extreme lengths under study.

The standard error of x_m computed from equation 4 is added to and subtracted from the length x_m found along the theoretical line to get the upper point and lower point of the control curves. Plot these two points parallel to the length axis since these are length values also. If only one σ unit is used to get the control curves, then there is the probability of 0.6827 (approximately 2/3) that each point is contained in the area enclosed by the two curves. If two σ units are used, the interval made by the control curves expands and the probability increases accordingly to 0.9545.

The control curves are used as a check on the amount of scatter of the extreme length values about the fitted line. In other words, they may be considered as confidence bands of the dispersion of observations about their theoretical values.

Analysis of the data related to control curves are safely made for $\Phi(y)$ values between .15 and .85, or else, errors may be encountered in interpretation.

5. Expected Extremes

An expected largest value, u_n, is defined as an extreme length that is expected to occur in a sample of size n with a probability given by:

$$F(u_n) = 1 - 1/n \qquad \text{(Equation 5)}$$

Of course, the expected largest value is not the mean largest value. Likewise, the probability of the expected smallest value is:

$$F(u_1) = 1/n$$

The expected extreme $x = u_n$ is obtained by first getting the corresponding reduced variate y for the probability $F(u_n)$ using equation 1. (Recall that $\Phi(y) = F(x)$). Then solve for u_n from equation 2 using the parameters u and $1/\alpha$ computed from the observations.

To determine the relationship of the expected largest value u_n and the sample size n, the quantity α_n is introduced and is defined as

$$\alpha_n = nf(u_n)$$

where $f(u_n) = F'(u_n)$ or may be computed using equation 1.

Taking the derivative of equation 5 with respect to n, the following is obtained:

$$\frac{d\,u_n}{d \log n} = \frac{1}{\alpha_n} \qquad \text{(Equation 6)}$$

Therefore, $1/\alpha_n$ measures the increase of u_n with the logarithm of n.

Equation 6 is called the <u>trend of logarithmic increase of the extremes</u> and is further stated as follows: If α_n is independent of n, u_n increases with log n. If α_n increases with n, u_n increases more slowly than log n. If α_n decreases with n, u_n increases more quickly than log n.

Therefore, the trend of logarithmic increase of the extremes determines whether expected extremes vary greatly with varying sample sizes.

IV. RESULTS

Length data from fishing vessels using purse seine and trawl net are dealt separately. (The data used have been provided by the Department of Agriculture and are part of their on-going fish stock assessment project).

A. PURSE SEINE (Refer to Figure 1).

The data set used in the application of the theory of extreme values consists of several fishing vessels using purse seine. The smallest and largest length are 18.5 cm and 28 cm, respectively.

There seems to be a good fit between the observed lengths and the theoretical line except for some points (18.5 cm and 24 cm) which deviate a little bit from the fitted straight line having the equation

$$y = .6124(x - 22.3810)$$

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and go outside the area covered by the control curves.

The computed expected extreme u_n for this data set with a sample size equal to 31 is 27.9617595628 cm. Taking the value of the quantity α_n for n = 31 and for other sample sizes, α_n increases with increasing n. Therefore, based on the trend of logarithmic increase of the extremes, the expected extreme u_n increases more slowly than log n. This means that increasing the sample size will also change the expected extreme but the rate of increase is rather slow to have big changes in interpretation.

With the value of u_n given above and taking its reduced variate y, the length interval covered by the control curves is (26.329 cm, 29.595 cm). Maximum length values obtained from other sources and such values arrived at using other procedures may be validated using this interval. Here, there is a probability of around 68% that the true maximum length of a R. brachysoma lies in the interval (26.329 cm, 29.595 cm).

B. TRAWL NET (Refer to Figure 2).

This data set contains maximum lengths from 131 surveys of fishing vessels using trawl net. As one may observe from Figure 2, the survey lengths don't greatly depart from the theoretical line

$$y = .6476(x - 21.9700).$$

If analysis is to be based on $\Phi(y)$ values from .15 to .85, the control curves could say that the above fitted line provides a good fit for the survey data. However, for larger lengths, the observed points go beyond the interval of the control curves and the fitted line.

The expected extreme u_n is computed as 29.49238247 cm and from this value, validation of maximum length values is based on the interval (27.9482 cm, 31.0366 cm).

C. A COMPARISON OF RESULTS

Other authors listed the following values for R. brachysoma:

Gulf of Thailand 1970 (Somjaiwong and Chullasorn)	18.20 cm
Gulf of Thailand 1970 (Sucondharman, et al)	19.60
Gulf of Thailand 1970 (Sucondharman, et al)	20.00
Gulf of Thailand 1972 (Hongskul)	20.90
Gulf of Thailand 1970 (Somjaiwong and Chullasorn)	20.90
Thailand West 1985 (Anonymous)	22.40
Java Sea 1979 (Dwiponggo, et al)	22.90
Gulf of Thailand 1970 (Kurogane)	23.00
Malaysia 1985 (Anonymous)	23.50
Thailand West 1985 (Anonymous)	24.50
Ragay Gulf 1981 (Corpuz, et al)	24.50
Samar Sea 1979-80 (Ingles and Pauly)	25.00
Samar Sea 1981 (Corpuz, et al)	25.00
Samar Sea 1979 (Corpuz, et al)	25.50
Thailand West 1985 (Anonymous)	26.30
Sumatra 1985 (Anonymous)	26.50
Manila Bay 1978-79 (Ingles and Pauly)	34.00

Comparing these values and the intervals obtained above -- (26.329 cm, 29.595 cm) for purse seine data and (27.948 cm, 31.037 cm) for trawl net data -- maximum lengths found in other sources (and using other methods) seem to be underestimated. Most of these measurements are relatively smaller than the lower limits of the intervals given here.

SUMMARY

The statistical theory of extreme values aims to explain the occurrence of far-removed observations and to predict extreme points that may occur. There is a wide field of interest over which this theory may be applied. In this article, the statistical theory of extreme values is applied to the maximum lengths of fish (obtained from catches of commercial fishing vessels) and with the hope of validating L_{max} values obtained by other authors.

If the initial sample distribution is known, the exact distribution of extremes may be easily obtained. If it isn't but the type of distribution is known, the asymptotic distribution of extremes may then be obtained. There are three types of asymptotic distributions available. They are the exponential type, the Cauchy type and the limited distribution. The exponential type is particularly discussed in this article since the two other asymptotic distributions may be transformed into this type.

A theoretical straight line, $x = u + y/\alpha$, is fitted to the ordered observations plotted on an extremal probability paper with the observed lengths on the vertical axis and the corresponding reduced variate y on the horizontal axis. The extremal probability paper graphically checks the goodness of fit between theoretical (or expected) length measurements and the actual observations. u is an average of the extreme value distribution and $1/\alpha$ is a measure of dispersion.

The parameters mentioned above are computed as follows:

$$1/\alpha = \frac{S_x}{\sigma_n} , \qquad u = \bar{x} - \bar{y}_n/\alpha$$

$\bar{x}$ and S_x are sample mean and sample standard deviation. $\bar{y}_n$ and σ_n are the expected (or reduced) mean and standard deviation obtained from the plotting position, m/(n+1).

Control curves are computed to check the amount of scatter and to determine the fit of the actual measurements about the fitted curve.

The expected extreme value for a sample size and its corresponding length interval covered by the control curves may be used to validate extreme length values obtained using other methods. Usually, the control curves are constructed with one unit to establish their distances from the theoretical line. This provides a 68% probability that the true maximum length lies in the length interval enclosed by the control curves for the computed expected extreme value.

In this article, the theory of extreme values was applied to two data sets of length values obtained using different gears. Computations revealed that there is a 68% probability that the true maximum length of a R. brachysoma lies in the interval (26.329 cm, 29.5 5 cm) for the purse seine data and (27.9482 cm, 31.0366 cm) for the trawl net data. Maximum lengths of this species found in the literature and using other methods seem to be underestimated in comparison.

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THE COMPUTER PROGRAM

The procedures discussed in this article may be conveniently performed using the computer program provided here. This program written in BASIC and made for use with an HP 86 computer provides instructions for the following: computation of the parameters u, $1/\alpha$, $\bar{y}_n$ and σ_n which are needed for computing the line of expected extremes (equation 2 or 2A) without using the table of these values in Gumbel's monograph; the construction of the line of expected extremes and control curves; the computation of expected extreme values; and a graphics section featuring the plotting of observed values, the theoretical line and the control curves.

This program provides the option of whether a hardcopy of the computed results and the graphics display is desired or not. It also allows the creation and retrieval of data and graphics files. However, all necessary data files must be arranged according to size before running the program.

FIGURES

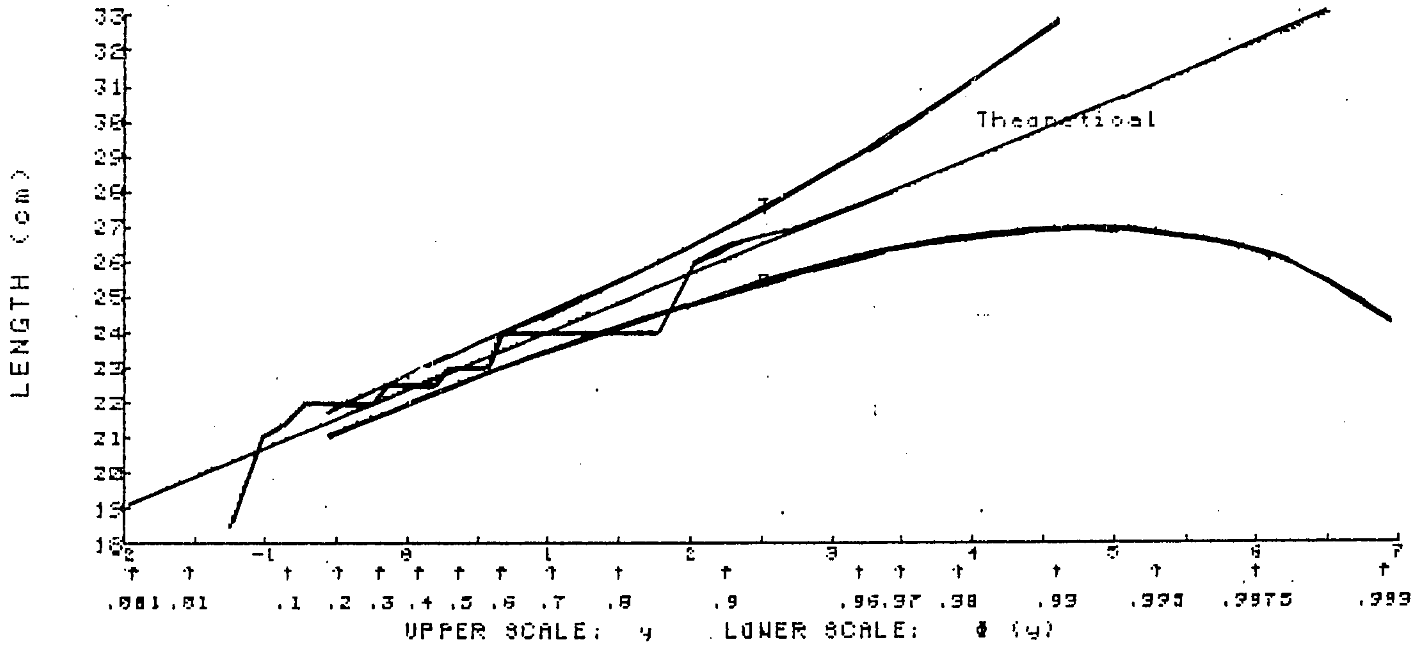

Figure 1. Maximum lengths of R. brachysoma caught using purse seine from the Visayan Sea, 1984.

COLOR CODE:

- observed values
- theoretical line
- control curves

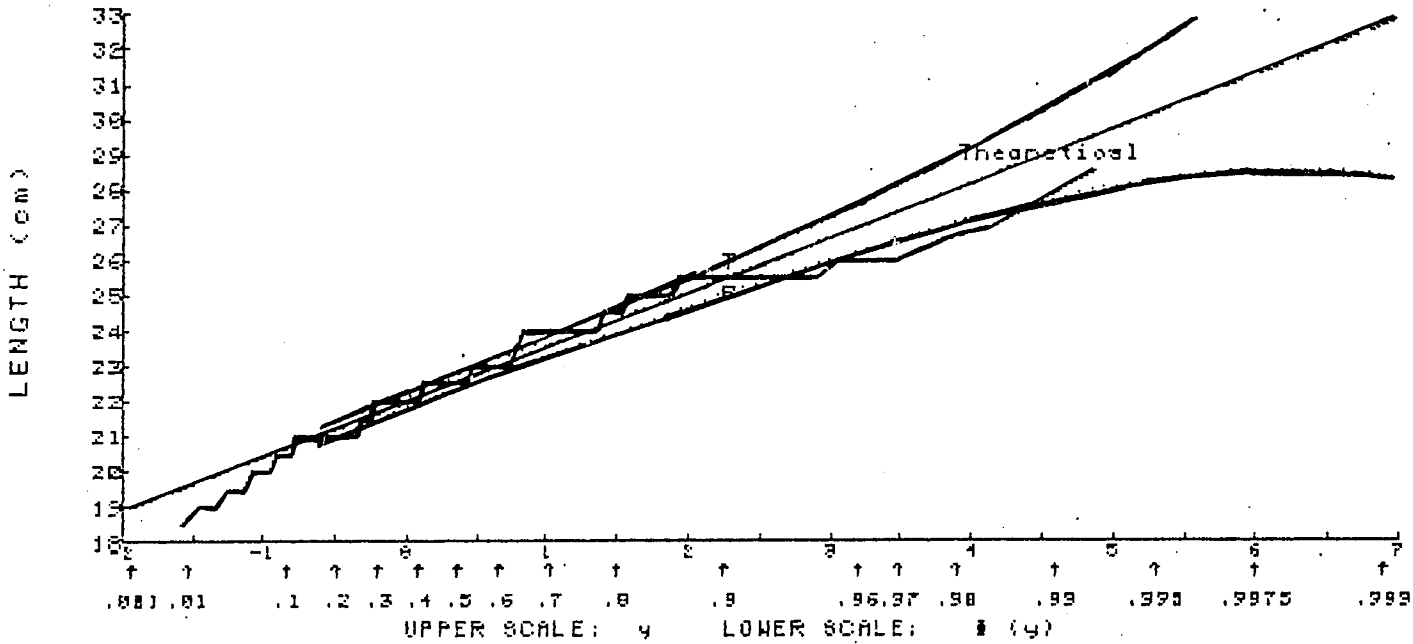

Figure 2. Maximum lengths of R. brachysoma caught using trawl net from the Visayan Sea, 1984

COLOR CODE:

- observed values
- theoretical line
- control curves

COMPUTER PROGRAM AND PRINTOUTS

```
10 REM :  THEORET3 - CRSP
20 REM : DATE OF LAST REVISION >> JANUARY 28, 1988
30 OPTION BASE 1
40 DIM OBS(500),P(500),Y(500),YY(2000)
50 DIM R(2000),B(2000),T(2000),MARKX(3),MARKY(3)
60 !
70 ! ********** DATA ENTRY *************
80 CLEAR
90 DISP "ENTER DATA FROM :"
100 DISP "     [K]eyboard"
110 DISP "     [D]iskette"
120 DISP "     [E]xit"
130 DISP @ DISP "SELECT OPTION ";@ INPUT OPT$
140 IF OPT$="K" THEN 260
150 IF OPT$="D" THEN 370
160 IF OPT$="E" THEN 3990
170 GOSUB ERROR
180 GOTO 80
190 !
200 ERROR:
210 CLEAR
220 DISP "ERROR !"
230 BEEP 10000,10000
240 RETURN
250 !
260 ! KEYBOARD DATA ENTRY
270 CLEAR @ RECALL=0
280 DISP "NO. OF OBSERVATIONS ";@ INPUT NOBS
290 IF NOBS<2 OR NOBS#INT (NOBS) THEN GOSUB ERROR  ELSE 310
300 GOTO 270
310 FOR I=1 TO NOBS
320    DISP "ENTER OBSERVATION# ";I;@ INPUT OBS(I)
330 NEXT I
340 GOTO 490
350 !
360 !
370 ! READ DATA FROM DISKETTE FILE
380 CLEAR @ RECALL=1
390 DISP "NAME OF FILE TO RECALL ";@ INPUT FILENAME$
400 ASSIGN# 1 TO  FILENAME$
410 FOR I=1 TO 500
420 READ# 1,I ; OBS(I)
430 IF OBS(I)#0 THEN 450
440 NOBS=I-1 @ I=500
450 NEXT I
460 GOSUB 490
470 GOTO 710
480 !
490 CLEAR
500 DISP "DO YOU WANT TO PRINT DATA (Y/N) ";@ INPUT Y$
510 IF Y$="Y" THEN 550
520 IF Y$="N" THEN 710
530 GOSUB ERROR
540 GOTO 490
550 GOSUB 570
560 GOTO 710
570 CLEAR
```

```
580 DISP "PRESS <CONT> WHEN PRINTER IS READY." @ PAUSE
590 PRINTER IS 708
600 IF NOT RECALL THEN 620
610 PRINT "FILE : ";FILENAME$
620 PRINT "(INDEX) OBSERVATION"
630 FOR I=1 TO NOBS
640 IF I=NOBS THEN 660
650 PRINT "(";VAL$ (I);")";OBS(I);", ";@ GOTO 670
660 PRINT "(";VAL$ (I);")";OBS(I)
670 NEXT I
680 FOR I=1 TO 7 @ PRINT @ NEXT I
690 RETURN
700 !
710 EDIT=0
720 CLEAR
730 DISP "ARE ALL ENTRIES CORRECT (Y/N) ";@ INPUT Y$
740 IF Y$="Y" THEN 890
750 IF Y$="N" THEN 790
760 GOSUB ERROR
770 GOTO 720
780 !
790 EDIT=1 @ CLEAR
800 DISP "INDEX OF OBSERVATION TO CORRECT ";@ INPUT INDEX
810 IF INDEX<1 OR INDEX>NOBS OR INDEX#INT (INDEX) THEN 790
820 DISP
830 DISP "INDEX";INDEX;" >> Previous entry :";OBS(INDEX)
840 DISP "ENTER NEW OBSERVATION ";
850 INPUT OBS(INDEX)
860 IF NOT RECALL THEN 880
870 PRINT# 1,INDEX ; OBS(INDEX)
880 GOTO 720
890 IF NOT EDIT THEN 970
900 CLEAR
910 DISP "DO YOU WANT TO PRINT EDITED FILE (Y/N) ";@ INPUT Y$
920 IF Y$="Y" THEN 960
930 IF Y$="N" THEN 970
940 GOSUB ERROR
950 GOTO 900
960 GOSUB 570
970 IF RECALL THEN 1140
980 CLEAR
990 DISP "DO YOU WANT TO SAVE ENTRIES (Y/N) ";@ INPUT Y$
1000 IF Y$="Y" THEN 1040
1010 IF Y$="N" THEN 1140
1020 GOSUB ERROR
1030 GOTO 980
1040 CLEAR
1050 DISP "NAME OF FILE TO CREATE ";@ INPUT FILENAME$
1060 OBS(NOBS+1)=0
1070 CREATE FILENAME$,NOBS+1,8
1080 ASSIGN# 1 TO FILENAME$
1090 FOR I=1 TO NOBS+1
1100 PRINT# 1,I ; OBS(I)
```

```
1110 NEXT I
1120 ASSIGN# 1 TO  *1130 !
1140 CLEAR
1145 DISP "Computing  n &  Y, please wait...." @ BEEP 10,100
1150 ! TO COMPUTE Yn AND  n
1160 K=0 @ Q=0
1170 FOR I=1 TO NOBS
1180 RR=I/(NOBS+1)
1190    SS=-LOG (-LOG (RR))
1200    K=K+SS
1210 Q=Q+SS^2
1220 NEXT I
1230 H=K/NOBS
1240 DA=SQR ((Q-K^2/NOBS)/NOBS)
1250 DISP "MEAN Y VALUE IS ",H
1260 !
1270 DISP " SIGMA n VALUE IS ",DA
1290 !
1300 DISP "PRINT ALL RESULTS (Y/N) ";@ INPUT Y$
1310 IF Y$="Y" THEN PRINTER IS 708 @ GOTO 1340
1320 IF Y$="N" THEN PRINTER IS 1 @ GOTO 1350
1330 GOSUB ERROR  @ GOTO 1300
1340 DISP "PRESS <CONT> WHEN PRINTER IS READY." @ PAUSE
1350 CLEAR @ DISP "COMPUTING, Please wait...." @ BEEP 10,1000
1360 REM : COMPUTE PLOTTING POSITION (P) AND REDUCED VARIATE (Y)
1370 PRINT
1380 PRINT TAB (9);"LENGTH";TAB (19);"PHI(X)";TAB (32);"Y"
1390 FOR I=1 TO NOBS
1400      P(I)=I/(NOBS+1)
1410      Y(I)=-LOG (-LOG (P(I)))
1420      PRINT USING 1440 ; I,OBS(I),P(I),Y(I)
1430 NEXT I
1440 IMAGE 3X,3D,2X,3D.D,5X,Z.4D,4X,DZ.4D
1450 !
1460 REM : COMPUTE MEAN AND STANDARD DEVIATION OF OBSERVED VALUES
1470 SUMM,SQUARE=0
1480 FOR I=1 TO NOBS
1490      SUMM=OBS(I)+SUMM
1500      SQUARE=OBS(I)^2+SQUARE
1510 NEXT I
1520 !
1530 AVE=SUMM/NOBS
1540 STANDEV=SQR ((SQUARE-SUMM^2/NOBS)/NOBS)
1550 !
1560 REM : COMPUTE 1/ALPHA AMD MU
1570 W=STANDEV/DA
1580 U=AVE-H*W
1590 PRINT
1600 PRINT
1610 PRINT "Yn = ";H
1620 PRINT "SIGMA n = ";DA
1630 PRINT "MEAN OF OBSERVED LENGTHS = ";AVE
1640 PRINT "STANDARD DEVIATION OF OBSERVED LENGTHS = ";STANDEV
1650 PRINT "1/Alpha = ";W
1660 PRINT "Mu      = ";U
1670 !
1680 REM : FIND THEORETICAL LINE AND CONTROL CURVES
```

```
1690 PRINT
1700 PRINT TAB (8);"LENGTH";TAB (17);"PHI(Y)";TAB (30);"LOW";TAB (40);"HIGH"
1710 IMAGE 3X,3D,2X,3D.D,3X,DZ.4D,3X,3D.4D,3X,3D.4D
1720 X=OBS(1) @ I=1
1730     YY(I)=W^-1*(X-U) !                       REDUCED VARIATE
1740     R(I)=EXP (-EXP (-YY(I))) !                  THEORETICAL CUMULATIVE FREQ
1750     J=EXP (-YY(I)-EXP (-YY(I))) !               FIRST DERIVATIVE OF R
1760     M=SQR (R(I)*(1-R(I)))/J !                  STANDARD ERROR#1
1770     L=M*W/SQR (NOBS) !                         STANDARD ERROR FOR CONTROL CURVES
1780     B(I)=X-L !                                 LOWER POINT (CONTROL CURVE)
1790     T(I)=X+L !                                 UPPER POINT (CONTROL CURVE)
1800     PRINT USING 1710 ; I,X,YY(I),B(I),T(I)
1810     IF YY(I)>7 THEN 1850
1820     X=X+.5 @ I=I+1
1830     GOTO 1730
1840 !
1850 LAST=I
1860 ! ********** EXPECTED EXTREMES **********
1870 BIGF=1-1/NOBS
1880 YEXT=-LOG (-LOG (BIGF))
1890 XEXT=U+YEXT*W
1900 SMALLF=EXP (-YEXT-EXP (-YEXT))
1910 SIGMA1=SQR (BIGF*(1-BIGF))/SMALLF
1920 SIGMACC=SIGMA1*W/SQR (NOBS)
1930 LOWER=XEXT-SIGMACC @ UPPER=XEXT+SIGMACC
1940 DISP "EXPECTED EXTREME VALUE IS ",XEXT
1950 DISP "LOWER POINT OF CONTROL CURVE FOR EXPECTED EXTREME IS ",LOWER
1960 DISP "UPPER POINT OF CONTROL CURVE FOR EXPECTED EXTREME IS ",UPPER
1970 DISP
1980 DISP "DO YOU WANT TO PRINT THESE RESULTS (Y/N) ";@ INPUT Y$
1990 IF Y$="Y" THEN PRINTER IS 708 @ GOTO 2020
2000 IF Y$="N" THEN 2050
2010 GOSUB ERROR
2012 PRINT @ PRINT
2020 PRINT "EXPECTED EXTREME VALUE = ";XEXT
2030 PRINT "LOWER POINT OF CONTROL CURVE FOR EXPECTED EXTREME = ";LOWER
2040 PRINT "UPPER POINT OF CONTROL CURVE FOR EXPECTED EXTREME = ";UPPER
2050 DISP @ DISP "Press <CONT> to proceed." @ PAUSE
2060 ! ********** PLOT CURVES *************
2070 !
2080 GRAPHALL
2090 LIMIT 0,171,0,75.2
2100 !
2110 LOCATE 20,222,15,97
2120 RAISE=1
2130 IF FP (OBS(1))=0 THEN 2150
2140 YMIN=OBS(1)-FP (OBS(1)) @ GOTO 2160
2150 YMIN=OBS(1)
2160 IF FP (OBS(NOBS))=0 THEN 2180
2170 YMAX=OBS(NOBS)-FP (OBS(NOBS))+5 @ GOTO 2190
2180 YMAX=OBS(NOBS)+5
2190 CUT=0 @ INC=1
2200 IF YMAX<100 THEN 2300
2210 CUT=1
2220 FOR I=10 TO 100000 STEP 10 !                  REDUCE BIG NUMBERS
```

```
2230 EXPCUT=YMAX/I
2240 IF EXPCUT>100 THEN 2260
2250 RAISE=I @ I=100000 !                                RAISE  >>>>  [No] * RAISE
2260 NEXT I
2270 YMAX=YMAX/RAISE @ INC=5 !                           SET Y-INCREMENT TO 5
2280 YMAX=INT (YMAX/10)*10+10 !                          NEW Y-MAXIMUM
2290 YMIN=0 !                                            NEW Y-MINIMUM
2300 SCALE -2,7,YMIN,YMAX
2310 XAXIS YMIN,.5,-2,7
2320 IF CUT THEN 2340
2330 YAXIS -2,1,YMIN,YMAX @ GOTO 2350
2340 YAXIS -2,INC,YMIN,YMAX
2350 !
2360 !
2370 !
2380 ! X-AXIS TIC LABELS
2390 LOCATE 20,222,0,15
24[redacted]0 SCALE -2,7,0,10
2[redacted] CSIZE 3 @ LORG 5
2420 FOR X=-2 TO 7
2430 MOVE X,9
2440 LABEL VAL$ (X)
2450 NEXT X
2460 DATA -1.93,.001,-1.53,.01,-.83,.1
2461 DATA -.48,.2,-.19,.3,.09,.4,.37,.5
2462 DATA .67,.6,1.03,.7,1.5,.8,2.25,.9
2463 DATA 3.2,.96,3.49,.97,3.9,.98,4.6,.99
2464 DATA 5.3,.995,6,.9975.6.9,.999
2470   CSIZE 3.5
2480   FOR I=1 TO 18
2490   READ A,B@ MOVE A,7 @ LABEL " "
2495   MOVE A,4 @ LABEL VAL$ (B)
2500   NEXT I
2510 !
2[redacted]0 !
2530 !
2840 !
2850 ! X-AXIS TITLE
2860 CSIZE 4 @ LORG 2
2870 MOVE 0,1
2880 LABEL "UPPER SCALE:  y    LOWER SCALE:   I (y) "
2890 !
2910 !
2920 ! Y-AXIS TIC LABELS
2930 LOCATE 20,222,15,97
2940 SCALE -2,7,YMIN,YMAX
2950 CSIZE 4
2960 FOR Y=YMIN TO YMAX STEP INC
2970 MOVE -2.2,Y
2980 LABEL VAL$ (Y)
2990 NEXT Y
3000 !
3010 ! Y-AXIS TITLE
```

```
3020 MID=(YMIN+YMAX)/2 !          FIND MIDDLE POINT OF Y-AXIS
3030 DEG @ CSIZE 5 !
3040 LORG 5 @ LDIR 90 !
3050 MOVE -2.75,MID
3060 LABEL "LENGTH (cm)"
3070 ! REDUCE DATA
3080 FOR I=1 TO NOBS
3090 OBS(I)=OBS(I)/RAISE
3100 NEXT I
3110 !
3120 !
3130 ! PLOT LENGTHS
3140 FOR I=1 TO NOBS
3150 IF I#1 THEN 3170
3160 MOVE Y(I),OBS(I) @ GOTO 3180
3170 DRAW Y(I),OBS(I)
3180 NEXT I
3190 LDIR 0 @ CSIZE 4
3200 ! PLOT THEORETICAL LINE
3210 X=OBS(1)*RAISE
3220 FOR JOANNA=1 TO LAST
3230 IF YY(JOANNA)<-2 THEN 3250
3240 START=JOANNA @ JOANNA=LAST @ GOTO 3260
3250 X=X+.5
3260 NEXT JOANNA
3270 OK=0
3280 LINE TYPE 5
3290 FOR I=START TO LAST
3300 IF I#START THEN 3320
3310 MOVE YY(I),X/RAISE @ GOTO 3360
3320 DRAW YY(I),X/RAISE
3330 IF YY(I)<4.5 THEN 3360
3340 IF OK THEN 3360
3350 MARKX(1)=YY(I) @ MARKY(1)=X/RAISE @ OK=1
3360 X=X+.5
3370 NEXT I
3380 !
3390 ! PLOT CONTROL CURVES
3400 ! - lower curve first
3410 X=OBS(1)
3420 FOR JOANNA=1 TO LAST
3430 IF R(JOANNA)<.15 THEN 3450
3440 FIRST=JOANNA @ JOANNA=LAST @ GOTO 3460
3450 X=X+.5
3460 NEXT JOANNA
3470 OK=0
3480 LINE TYPE 3
3490 FOR I=FIRST TO LAST
3500 IF I#FIRST THEN 3520
3510 MOVE YY(I),B(I)/RAISE @ GOTO 3560
3520 DRAW YY(I),B(I)/RAISE
3530 IF R(I)<.9 THEN 3570
3540 IF OK THEN 3560
3550 MARKX(2)=YY(I) @ MARKY(2)=B(I)/RAISE @ OK=1
```

```
3560 X=X+.5
3570 NEXT I
3580 OK=0
3590 ! - upper curve
3600 FOR I=FIRST TO LAST
3610 IF I#FIRST THEN 3630
3620 MOVE YY(I),T(I)/RAISE @ GOTO 3670
3630 DRAW YY(I),T(I)/RAISE
3640 IF R(I)<.9 THEN 3680
3650 IF OK THEN 3670
3660 MARKX(3)=YY(I) @ MARKY(3)=T(I)/RAISE @ OK=1
3670 X=X+.5
3680 NEXT I
3690 !
3700 ! LABEL PLOTS
3710 FOR I=1 TO 3
3720 READ A$
3730 MOVE MARKX(I),MARKY(I) @ LABEL A$
3740 NEXT I
3750 DATA "Theoretical","B","T"
3760 GSTORE "GRAFSAVE"
3770 ALPHA
3780 CLEAR @ BEEP
3790 DISP "DO YOU WANT A HARD-COPY OF THE GRAPH (Y/N) ";@ INPUT Y$
3800 IF Y$="Y" THEN 3830
3810 IF Y$="N" THEN 3840
3820 GOSUB ERROR  @ GOTO 3780
3830 DUMPGRAF=1 @ GOTO 3850
3840 DUMPGRAF=0
3850 CLEAR @ PRINTER IS 708
3860 DISP "Please wait...." @ BEEP 10,100
3870 GLOAD "GRAFSAVE"
3880 IF NOT DUMPGRAF THEN 3910
3890 DUMP GRAPHICS 0,0,0,1
3900 GOTO 3940
3910 LOCATE 20,222,0,25
3920 SCALE 1,10,1,10
3930 MOVE 3,8 @ LABEL ">>> PRESS <CONT> TO PROCEED." @ PAUSE
3940 ALPHA @ CLEAR
3950 !
3960 BEEP 10000,10000
3970 RESTORE @ GOTO 70
3980 !
3990 REM : END PROGRAM
4000 GLOAD "VCSJ"
4010 BEEP 10,1000
4020 END
```

FILE : R brach
(INDEX) OBSERVATION
(1) 18.5 , (2) 21 , (3) 21.5 , (4) 22 , (5) 22 , (6) 22 , (7) 22 , (8) 22 , (9) 22 , (10) 22.5 , (11) 22.5 , (12) 22.5 , (13) 22.5 , (14) 22.5 , (15) 23 , (16) 23 , (17) 23 , (18) 23 , (19) 24 , (20) 24 , (21) 24 , (22) 24 , (23) 24 , (24) 24 , (25) 24 , (26) 24 , (27) 24 , (28) 26 , (29) 26.5 , (30) 27 , (31) 28

	LENGTH	PHI(X)	Y
1	18.5	0.0313	-1.2429
2	21.0	0.0625	-1.0198
3	21.5	0.0938	-0.8617
4	22.0	0.1250	-0.7321
5	22.0	0.1563	-0.6186
6	22.0	0.1875	-0.5152
7	22.0	0.2188	-0.4186
8	22.0	0.2500	-0.3266
9	22.0	0.2813	-0.2378
10	22.5	0.3125	-0.1511
11	22.5	0.3438	-0.0656
12	22.5	0.3750	0.0194
13	22.5	0.4063	0.1045
14	22.5	0.4375	0.1903
15	23.0	0.4688	0.2775
16	23.0	0.5000	0.3665
17	23.0	0.5313	0.4580
18	23.0	0.5625	0.5528
19	24.0	0.5938	0.6514
20	24.0	0.6250	0.7550
21	24.0	0.6563	0.8646
22	24.0	0.6875	0.9816
23	24.0	0.7188	1.1079
24	24.0	0.7500	1.2459
25	24.0	0.7813	1.3989
26	24.0	0.8125	1.5720
27	24.0	0.8438	1.7726
28	26.0	0.8750	2.0134
29	26.5	0.9063	2.3183
30	27.0	0.9375	2.7405
31	28.0	0.9688	3.4499

$\bar{Y}n$ = .537127875994
SIGMA n = 1.11591681014
MEAN OF OBSERVED LENGTHS = 23.2580645161
STANDARD DEVIATION OF OBSERVED LENGTHS = 1.82222377348
1/Alpha = 1.63293872529
Mu = 22.380967607

	LENGTH	PHI(Y)	LOW	HIGH
1	18.5	-2.3767	12.5629	24.4371
2	19.0	-2.0705	17.0515	20.9485
3	19.5	-1.7643	18.5710	20.4290
4	20.0	-1.4581	19.4186	20.5812
5	20.5	-1.1519	20.0587	20.9413
6	21.0	-0.8457	20.6166	21.3834
7	21.5	-0.5395	21.1350	21.8650
8	22.0	-0.2333	21.6302	22.3698
9	22.5	0.0729	22.1093	22.8907
10	23.0	0.3791	22.5752	23.4248
11	23.5	0.6853	23.0289	23.9711
12	24.0	0.9915	23.4702	24.5298
13	24.5	1.2977	23.8983	25.1017
14	25.0	1.6039	24.3117	25.6883
15	25.5	1.9101	24.7087	26.2913
16	26.0	2.2163	25.0870	26.9130
17	26.5	2.5225	25.4436	27.5564
18	27.0	2.8287	25.7755	28.2245
19	27.5	3.1349	26.0785	28.9215
20	28.0	3.4411	26.3481	29.6519
21	28.5	3.7473	26.5789	30.4211
22	29.0	4.0534	26.7645	31.2355
23	29.5	4.3596	26.8977	32.1023
24	30.0	4.6658	26.9697	33.0303
25	30.5	4.9720	26.9706	34.0294
26	31.0	5.2782	26.8885	35.1115
27	31.5	5.5844	26.7099	36.2901
28	32.0	5.8906	26.4189	37.5811
29	32.5	6.1968	25.9967	39.0033
30	33.0	6.5030	25.4218	40.5782
31	33.5	6.8092	24.6690	42.3310
32	34.0	7.1154	23.7087	44.2913

EXPECTED EXTREME VALUE = 27.9617595628
LOWER POINT OF CONTROL CURVE FOR EXPECTED EXTREME = 26.3287476828
UPPER POINT OF CONTROL CURVE FOR EXPECTED EXTREME = 29.5947714429

FILE : REVTRAWL
(INDEX) OBSERVATION
(1) 18.5 , (2) 19 , (3) 19 , (4) 19.5 , (5) 19.5 , (6) 19.5 , (7) 20 , (8) 20 ,
(9) 20 , (10) 20 , (11) 20.5 , (12) 20.5 , (13) 20.5 , (14) 20.5 , (15) 21 , (16
) 21 , (17) 21 , (18) 21 , (19) 21 , (20) 21 , (21) 21 , (22) 21 , (23) 21 , (24
) 21 , (25) 21 , (26) 21 , (27) 21 , (28) 21 , (29) 21 , (30) 21 , (31) 21 , (32
) 21 , (33) 21.5 , (34) 21.5 , (35) 21.5 , (36) 21.5 , (37) 22 , (38) 22 , (39)
22 , (40) 22 , (41) 22 , (42) 22 , (43) 22 , (44) 22 , (45) 22 , (46) 22 , (47)
22 , (48) 22 , (49) 22 , (50) 22 , (51) 22 , (52) 22 , (53) 22 , (54) 22.5 , (
55) 22.5 , (56) 22.5 , (57) 22.5 , (58) 22.5 , (59) 22.5 , (60) 22.5 , (61)
22.5 , (62) 22.5 , (63) 22.5 , (64) 22.5 , (65) 22.5 , (66) 22.5 , (67) 22.5 ,
(68) 22.5 , (69) 22.5 , (70) 23 , (71) 23 , (72) 23 , (73) 23 , (74) 23 , (75)
23 , (76) 23 , (77) 23 , (78) 23 , (79) 23 , (80) 23 , (81) 23 , (82) 23 , (83)
23.5 , (84) 23.5 , (85) 24 , (86) 24 , (87) 24 , (88) 24 , (89) 24 , (90) 24 ,
(91) 24 , (92) 24 , (93) 24 , (94) 24 , (95) 24 , (96) 24 , (97) 24 , (98) 24 ,
(99) 24 , (100) 24 , (101) 24 , (102) 24 , (103) 24.5 , (104) 24.5 , (105) 24.5
, (106) 24.5 , (107) 25 , (108) 25 , (109) 25 , (110) 25 , (111) 25 , (112) 25
, (113) 25 , (114) 25.5 , (115) 25.5 , (116) 25.5 , (117) 25.5 , (118) 25.5 , (
119) 25.5 , (120) 25.5 , (121) 25.5 , (122) 25.5 , (123) 25.5 , (124) 25.5 , (
1[illegible]) 25.5 , (126) 26 , (127) 26 , (128) 26 , (129) 26.5 , (130) 27 , (131) 28.5

	LENGTH	PHI(X)	Y
1	18.5	0.0076	-1.5857
2	19.0	0.0152	-1.4326
3	19.0	0.0227	-1.3308
4	19.5	0.0303	-1.2518
5	19.5	0.0379	-1.1858
6	19.5	0.0455	-1.1285
7	20.0	0.0530	-1.0774
8	20.0	0.0606	-1.0308
9	20.0	0.0682	-0.9879
10	20.0	0.0758	-0.9479
11	20.5	0.0833	-0.9102
12	20.5	0.0909	-0.8746
13	20.5	0.0985	-0.8406
14	20.5	0.1061	-0.8081
15	21.0	0.1136	-0.7769
16	21.0	0.1212	-0.7468
17	21.0	0.1288	-0.7176
18	21.0	0.1364	-0.6894
19	21.0	0.1439	-0.6618
20	21.0	0.1515	-0.6350
21	21.0	0.1591	-0.6088
22	21.0	0.1667	-0.5832
23	21.0	0.1742	-0.5581
24	21.0	0.1818	-0.5334
25	21.0	0.1894	-0.5092

26	21.0	0.1970	-0.4853
27	21.0	0.2045	-0.4618
28	21.0	0.2121	-0.4386
29	21.0	0.2197	-0.4157
30	21.0	0.2273	-0.3931
31	21.0	0.2348	-0.3707
32	21.0	0.2424	-0.3486
33	21.5	0.2500	-0.3266
34	21.5	0.2576	-0.3049
35	21.5	0.2652	-0.2833
36	21.5	0.2727	-0.2618
37	22.0	0.2803	-0.2405
38	22.0	0.2879	-0.2193
39	22.0	0.2955	-0.1982
40	22.0	0.3030	-0.1772
41	22.0	0.3106	-0.1563
42	22.0	0.3182	-0.1355
43	22.0	0.3258	-0.1148
44	22.0	0.3333	-0.0940
45	22.0	0.3409	-0.0734
46	22.0	0.3485	-0.0527
47	22.0	0.3561	-0.0321
48	22.0	0.3636	-0.0115
49	22.0	0.3712	0.0091
50	22.0	0.3788	0.0297
51	22.0	0.3864	0.0503
52	22.0	0.3939	0.0709
53	22.0	0.4015	0.0916
54	22.5	0.4091	0.1123
55	22.5	0.4167	0.1330
56	22.5	0.4242	0.1538
57	22.5	0.4318	0.1747
58	22.5	0.4394	0.1956
59	22.5	0.4470	0.2166
60	22.5	0.4545	0.2377
61	22.5	0.4621	0.2589
62	22.5	0.4697	0.2802
63	22.5	0.4773	0.3016
64	22.5	0.4848	0.3231
65	22.5	0.4924	0.3447
66	22.5	0.5000	0.3665
67	22.5	0.5076	0.3884
68	22.5	0.5152	0.4105
69	22.5	0.5227	0.4328
70	23.0	0.5303	0.4552
71	23.0	0.5379	0.4778
72	23.0	0.5455	0.5007
73	23.0	0.5530	0.5237
74	23.0	0.5606	0.5469
75	23.0	0.5682	0.5704

76	23.0	0.5758	0.5941
77	23.0	0.5833	0.6180
78	23.0	0.5909	0.6423
79	23.0	0.5985	0.6668
80	23.0	0.6061	0.6916
81	23.0	0.6136	0.7167
82	23.0	0.6212	0.7422
83	23.5	0.6288	0.7680
84	23.5	0.6364	0.7941
85	24.0	0.6439	0.8206
86	24.0	0.6515	0.8476
87	24.0	0.6591	0.8749
88	24.0	0.6667	0.9027
89	24.0	0.6742	0.9310
90	24.0	0.6818	0.9597
91	24.0	0.6894	0.9890
92	24.0	0.6970	1.0188
93	24.0	0.7045	1.0492
94	24.0	0.7121	1.0803
95	24.0	0.7197	1.1119
96	24.0	0.7273	1.1443
97	24.0	0.7348	1.1774
98	24.0	0.7424	1.2112
99	24.0	0.7500	1.2459
100	24.0	0.7576	1.2815
101	24.0	0.7652	1.3180
102	24.0	0.7727	1.3555
103	24.5	0.7803	1.3940
104	24.5	0.7879	1.4338
105	24.5	0.7955	1.4747
106	24.5	0.8030	1.5170
107	25.0	0.8106	1.5608
108	25.0	0.8182	1.6061
109	25.0	0.8258	1.6531
110	25.0	0.8333	1.7020
111	25.0	0.8409	1.7529
112	25.0	0.8485	1.8060
113	25.0	0.8561	1.8617
114	25.5	0.8636	1.9200
115	25.5	0.8712	1.9814
116	25.5	0.8788	2.0463
117	25.5	0.8864	2.1150
118	25.5	0.8939	2.1882
119	25.5	0.9015	2.2665
120	25.5	0.9091	2.3506
121	25.5	0.9167	2.4417
122	25.5	0.9242	2.5411
123	25.5	0.9318	2.6505
124	25.5	0.9394	2.7723
125	25.5	0.9470	2.9098
126	26.0	0.9545	3.0679
127	26.0	0.9621	3.2541
128	26.0	0.9697	3.4812
129	26.5	0.9773	3.7727
130	27.0	0.9848	4.1820
131	28.5	0.9924	4.8790

$\bar{Y}_n$ = .563225536804
SIGMA n = 1.21958647204
MEAN OF OBSERVED LENGTHS = 22.8396946565
STANDARD DEVIATION OF OBSERVED LENGTHS = 1.88330063477
1/Alpha = 1.54421246705
Mu = 21.9699547608

	LENGTH	PHI(Y)	LOW	HIGH
1	18.5	-2.2471	16.8843	20.1157
2	19.0	-1.9233	18.3966	19.6034
3	19.5	-1.5995	19.1772	19.8228
4	20.0	-1.2757	19.7774	20.2226
5	20.5	-0.9519	20.3171	20.6829
6	21.0	-0.6281	20.8309	21.1691
7	21.5	-0.3043	21.3311	21.6689
8	22.0	0.0195	21.8224	22.1776
9	22.5	0.3432	22.3067	22.6933
10	23.0	0.6670	22.7847	23.2153
11	23.5	0.9908	23.2563	23.7437
12	24.0	1.3146	23.7212	24.2788
13	24.5	1.6384	24.1784	24.8216
14	25.0	1.9622	24.6271	25.3729
15	25.5	2.2860	25.0659	25.9341
16	26.0	2.6098	25.4932	26.5068
17	26.5	2.9336	25.9072	27.0928
18	27.0	3.2574	26.3056	27.6944
19	27.5	3.5811	26.6858	28.3142
20	28.0	3.9049	27.0446	28.9554
21	28.5	4.2287	27.3782	29.6218
22	29.0	4.5525	27.6824	30.3176
23	29.5	4.8763	27.9520	31.0480
24	30.0	5.2001	28.1809	31.8191
25	30.5	5.5239	28.3620	32.6380
26	31.0	5.8477	28.4870	33.5130
27	31.5	6.1715	28.5460	34.4540
28	32.0	6.4952	28.5274	35.4726
29	32.5	6.8190	28.4175	36.5825
30	33.0	7.1428	28.2004	37.7996

EXPECTED EXTREME VALUE = 29.492382473
LOWER POINT OF CONTROL CURVE FOR EXPECTED EXTREME = 27.9481662278
UPPER POINT OF CONTROL CURVE FOR EXPECTED EXTREME = 31.0365987182